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TEMPERATURE DISTRIBUTION IN A SPACE CONTAINING HEATED SPHERICAL INCLUSIONS PERIODICALLY ARRANGED ALONG A STRAIGHT LINE

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Consideration has been given to the problem for the Laplace equation in a bed with a spherical hole, on whose surface a homogeneous Dirichlet condition is formed, with periodicity conditions on the bed's planes and asymptotic conditions at infinity, which assign a logarithmic growth to the solution. Solution of the problem has been reduced to an infinite system of linear algebraic equations for the coefficients of periodic singular solutions of the Laplace equation with a singularity at the center of the hole.

1. Formulation of the Problem. Let $G = \mathbb{B}_l(O)$ be a sphere with center at the origin and radius l. We introduce the sets

$$G_\varepsilon = \left\{ \mathbf{x} \in \mathbb{R}^3 : \varepsilon^{-1} \mathbf{x} \in G \right\}, \quad Q_\varepsilon = (\mathbb{R}^2 \times (-l, l)) \setminus \overline{G}_\varepsilon ,$$

dependent on the small parameter $\varepsilon \in (0, 1]$. The region Q_ε represents a bed of thickness $2l$ with hole G_ε and a boundary $g_\varepsilon = \partial G_\varepsilon$.

In the region Q_ε, we consider the problem

$$\Delta_x e(\varepsilon; \mathbf{x}) = 0 , \quad \mathbf{x} = (\mathbf{y}, z) \in Q_\varepsilon ; \quad e(\varepsilon; \mathbf{x}) = 0 , \quad \mathbf{x} \in g_\varepsilon ; \tag{1}$$

$$e(\varepsilon; \mathbf{y}, -l) = e(\varepsilon; \mathbf{y}, -l) , \quad \frac{\partial e}{\partial z}(\varepsilon; \mathbf{y}, -l) = \frac{\partial e}{\partial z}(\varepsilon; \mathbf{y}, l) , \quad \mathbf{y} \in \mathbb{R}^2 ; \tag{2}$$

$$e(\varepsilon; \mathbf{x}) = -\ln |\mathbf{y}| + O(1) , \quad |\mathbf{y}| \to \infty . \tag{3}$$

The solvability of problem (1)–(3) was investigated in [1] (see Theorem 1 and Remark 1. *ii*). In view of the equality

$$-\frac{1}{4\pi l} \iint\limits_{g_\varepsilon} \frac{\partial e}{\partial n_x}(\varepsilon; \mathbf{x})\, d\sigma_x = 1 , \tag{4}$$

where n_x is the external (in relation to the region G_ε) normal to the surface g_ε, and the even parity (symmetry) of the function $e(\varepsilon;\ \mathbf{y},\ z)$ in the variable z problem (1)–(3) describes the temperature field in a bed with heat-insulated surfaces that contains a spherically shaped inclusion heated to a constant temperature. Formula (4) determines the value of the heat flux coming through the surface g_ε and radiated to infinity (see the asymptotic formula (3)). In the case of a region G_ε different from a spherical one the periodicity conditions (2) make it possible to interpret the solution of problem (1)–(3) as the temperature distribution in a space containing inclusions periodically arranged along the z axis and heated to a constant temperature.

We consider the formula refining the asymptotic expansion (3):

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$$e(\varepsilon;\mathbf{x}) = -\ln|\mathbf{y}| + \kappa(\varepsilon) + O(|\mathbf{y}|^{-1}), \quad |\mathbf{y}| \to \infty . \tag{5}$$

The quantity

$$c_{\log}(\overline{G}_\varepsilon) = \exp(\kappa(\varepsilon)) \tag{6}$$

is called the reduced logarithmic capacity [1].

We investigate the behavior of the quantity $c_{\log}(\overline{G}_\varepsilon)$ when $\varepsilon \to 0$ and, in particular, obtain the solution of problem (1)–(3).

2. Construction of the Solution. The presence of axial symmetry or symmetry about the plane $z = 0$ in the solution of problem (1)–(3) substantially simplifies construction of the solution.

We determine the harmonic functions satisfying the periodicity conditions (2) and having singularities at the points $z = \pm 2jl$ ($j = 0, 1, 2, \ldots$). We set

$$G^{2m}(\mathbf{x}) = \frac{P_{2m}(\cos\theta)}{r^{2m+1}} + \sum_{j=1}^{\infty} \frac{P_{2m}(\cos\theta_j)}{r_j^{2m+1}} + \frac{P_{2m}(\cos\theta_{-j})}{r_{-j}^{2m+1}} \quad (m = 1, 2, \ldots) . \tag{7}$$

Here $P_{2m}(\mu)$ is the Legendre polynomial of $2m$th order; we introduce the notation

$$r^2 = |\mathbf{y}|^2 + z^2, \quad |\mathbf{y}|^2 = y_1^2 + y_2^2, \quad \cos\theta = z/r ,$$

$$r_{\pm j}^2 = |\mathbf{y}|^2 + (z \mp 2jl)^2, \quad \cos\theta_{\pm j} = \frac{z \mp 2jl}{r_{\pm j}} .$$

When $|z| \le l$ the functional series in the representation (7) converges uniformly since it has a convergent (according to the Cauchy integral criterion) majorizing number series $2l^{-2m-1} \sum_{j=1}^{\infty} (2j-1)^{-2m-1}$.

When $m = 0$ the series in formula (7) diverges. Therefore, we set

$$G^0(\mathbf{x}) = \frac{1}{r} + \sum_{j=1}^{\infty} \frac{1}{r_j} + \frac{1}{r_{-j}} - \frac{2}{2jl} . \tag{8}$$

It is noteworthy that the coefficients of the series in formula (8) have the order j^{-2} for $j \to \infty$.

The solution of problem (1)–(3) will be represented in the form

$$e(\varepsilon;\mathbf{x}) = B_0 + lG(\mathbf{x}) + \sum_{m=1}^{\infty} B_m l^{2m+1} G^{2m}(\mathbf{x}) , \tag{9}$$

where B_0, B_1, ... are the coefficients to be determined that depend on the parameter ε.

We assume ourselves that the function (9) satisfies the asymptotic condition (3). For the sake of brevity we introduce the notation $f(j, \mathbf{x}) = \Sigma_\pm\, l(|\mathbf{y}|^2 + (z \mp 2jl)^2)^{-1/2}$. The representation

$$\sum_{j=1}^{n} f(j,\mathbf{x}) = \int_1^n f(t,\mathbf{x})\, dt + R_n(\mathbf{x}) \tag{10}$$

holds true (see, for example, [2], Chapter 8, Sec. 3). The remainder $R_n(\mathbf{x})$ is determined by the formula

$$R_n(\mathbf{x}) = \frac{1}{2} f(1, \mathbf{x}) + \frac{1}{2} f(n, \mathbf{x}) - 2l^2 \sum_{\pm} \int_1^n \frac{\omega_1(t)\,(2tl \mp z)\,dt}{(|\mathbf{y}|^2 + (2tl \mp z)^2)^{3/2}},$$

where $\omega_1(t) = t - [t] - (1/2)$ is the "sawtooth" function.

We use the known expansion (see, for example, [2], Chapter 8, Sec. 3)

$$\sum_{j=1}^{n} \frac{1}{j} = \int_1^n \frac{dt}{t} + \mathbf{C} + O(n^{-1}), \quad n \to \infty; \tag{11}$$

here $\mathbf{C} = 0.577 \ldots$.

Combining relations (10) and (11) and passing to the limit for $n \to \infty$, we obtain

$$\sum_{j=1}^{n} f(j, \mathbf{x}) - \frac{1}{j} = \int_1^{\infty} \left(f(t, \mathbf{x}) - \frac{1}{t} \right) dt - \mathbf{C} + R_{\infty}(\mathbf{x}) .$$

Thus, for the function (8) we find the following asymptotic representation:

$$lG^0(\mathbf{x}) = -\ln \frac{|\mathbf{y}|}{4l} - \mathbf{C} + O(|\mathbf{y}|^{-1}), \quad |\mathbf{y}| \to \infty . \tag{12}$$

Let us show now that the behavior of the function (9) at infinity (for $|\mathbf{y}| \to \infty$) is determined by the term $lG^0(\mathbf{x})$ and does not depend on the terms $B_m l^{2m+1} G^{2m}(\mathbf{x})$. For this purpose we consider the asymptotics (for $\rho \to \infty$) of the sum

$$S_{2m}(\rho) = \sum_{n=-\infty}^{+\infty} \frac{1}{(\rho^2 + n^2)^{m+\frac{1}{2}}} . \tag{13}$$

Following [3] and applying the summation formula of Poisson [see [3], Sec. 1.4) to the series (13), we find

$$S_{2m}(\rho) = \sum_{n=-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{\cos 2\pi nt}{(t^2 + \rho^2)^{m+\frac{1}{2}}}\, dt . \tag{14}$$

The integral (14) for $n = 0$ is equal to $b_m \rho^{-2m}$, where

$$b_m = 2 \int_0^{\infty} \frac{d\tau}{(\tau^2 + 1)^{m+\frac{1}{2}}} .$$

Expressing it (for $n = 1, 2, \ldots$) by the Macdonald function, we obtain

$$S_{2m}(\rho) = \frac{b_m}{\rho^{2m}} + 2 \sum_{n=1}^{\infty} \frac{(2\pi n)^m \Gamma\left(\frac{1}{2}\right)}{\Gamma\left(m + \frac{1}{2}\right)(2\rho)^m} K_m(2\pi n \rho) .$$

Finally, we use the asymptotic formula (see, for example, [4], Sec. 5.11)

$$K_m(x) = \left(\frac{\pi}{2x}\right)^{1/2} \exp\{-x\}\left[\sum_{j=0}^{k} \frac{(m, j)}{(2x)^j} + O(x^{-2k-2})\right], \quad (m, j) = \frac{\Gamma\left(m + \frac{1}{2} + j\right)}{j!\,\Gamma\left(m + \frac{1}{2} - j\right)} .$$

Thus, when $\rho \to +\infty$ the asymptotic relation

$$S_{2m}(\rho) - \frac{b_m}{\rho^{2m}} \sim 2\sqrt{\pi}\exp\{-2\pi\rho\} \sum_{j=0}^{\infty} \frac{(m, j)}{(4\pi\rho)^{j+\frac{1}{2}}} \tag{15}$$

holds true for the sum (13).

Noting that the inequalities $|P_{2m}(\mu)| \le 1$ occur when $-1 \le \mu \le 1$ and $|\mathbf{y}| = \rho < r_{\pm j}$ ($j = 1, 2, \ldots$), from formula (12) with account for (15) we derive for the function (9) the following asymptotic representation:

$$e(\varepsilon; \mathbf{x}) = -\ln|\mathbf{y}| + \ln 4l - \mathbf{C} + B_0 + O(|\mathbf{y}|^{-1}), \quad |\mathbf{y}| \to \infty . \tag{16}$$

Accordingly, comparing the expansions (16) and (15), for the reduced logarithmic capacity of a sphere $\overline{G}_\varepsilon$ we obtain the expression

$$c_{\log}(\overline{G}_\varepsilon) = 4l \exp(B_0 - \mathbf{C}) . \tag{17}$$

Thus, the function (9) completely satisfies the periodicity conditions (2) and the asymptotic condition (3) in construction, being harmonic everywhere in space except for the points $(0, 0, \pm 2jl)$ ($j = 0, 1, 2, \ldots$).

3. Transformation of Spherical Functions upon a Shift of the Origin of Coordinates. Following [5] (see Chapter 4, Sec. 7), we use the representation

$$\frac{P_n(\cos\theta_{\pm j})}{r_{\pm j}^{n+1}} = \frac{(-1)^n}{n!} \frac{\partial^n}{\partial z_\pm^n} \frac{1}{\sqrt{|\mathbf{y}|^2 + z_\pm^2}} ,$$

where $z_\pm = z \mp 2jl$. It is easily seen that

$$\frac{P_n(\cos\theta_{\pm j})}{r_{\pm j}^{n+1}} = \frac{(-1)^n}{n!} \frac{\partial^n}{\partial z^n} \frac{1}{\sqrt{|\mathbf{y}|^2 + (z \mp 2jl)^2}} .$$

Then we have

$$\frac{P_n(\cos\theta_{-j})}{r_{-j}^{n+1}} = \frac{(-1)^n}{n!\,(2l)^n} \frac{\partial^n}{\partial j^n} \frac{1}{\sqrt{|\mathbf{y}|^2 + (z + 2jl)^2}} , \tag{18}$$

$$\frac{P_n(\cos\theta_{j})}{r_{j}^{n+1}} = \frac{1}{n!\,(2l)^n} \frac{\partial^n}{\partial j^n} \frac{1}{\sqrt{|\mathbf{y}|^2 + (z - 2jl)^2}} . \tag{19}$$

Using now an expansion that holds true for $r < 2l$:

$$\frac{1}{r_{\pm j}} = \frac{1}{2jl}\left(1 \mp 2\frac{z}{2jl} + \frac{r^2}{(2jl)^2}\right)^{-1/2} = \frac{1}{2jl}\sum_{k=0}^{\infty} (\pm 1)^k P_k(\cos\theta)\left(\frac{r}{2jl}\right)^k ,$$

we find

$$\frac{P_n(\cos\theta_{\pm j})}{r_{\pm j}^{n+1}} = \frac{(\pm 1)^n}{n!\,(2l)^n} \frac{\partial^n}{\partial j^n} \sum_{k=0}^{\infty} (\pm 1)^k P_k(\cos\theta) \frac{r^k j^{-k-1}}{(2jl)^{k+1}} .$$

As a result of differentiation, we obtain

$$\frac{P_n(\cos\theta_{\pm j})}{r_{\pm j}^{n+1}} = \frac{(\mp 1)^n}{n!} \sum_{k=0}^{\infty} (\pm 1)^k \frac{(n+k)!}{k!} \frac{r^k}{(2jl)^{n+k+1}} P_k(\cos\theta) . \tag{20}$$

Relation (20) in the case of the representation (18) immediately follows from the transformation formula obtained in [5]. However, the case of the representation (19) is not reduced to the previous case by simple replacement of the subscript j by $-j$.

Thus, for $n = 2m$, from formula (20) we find

$$\sum_{\pm} \frac{P_{2m}(\cos\theta_{\pm j})}{r_{\pm j}^{2m+1}} = \frac{2}{m!} \sum_{s=0}^{\infty} \frac{(m+s)!}{s!} \frac{r^{2s}}{(2jl)^{2m+2s+1}} P_{2s}(\cos\theta) . \tag{21}$$

Using formula (21), we derive, according to the determinations (7) and (8), the following expressions:

$$G^0(\mathbf{x}) = \frac{1}{r} + 2 \sum_{j=1}^{\infty} \sum_{s=0}^{\infty} \frac{r^{2s}}{(2jl)^{2s+1}} P_{2s}(\cos\theta) , \tag{22}$$

$$G^{2m}(\mathbf{x}) = \frac{P_{2m}(\cos\theta)}{r^{2m+1}} + \frac{2}{m!} \sum_{j=1}^{\infty} \sum_{s=0}^{\infty} \frac{(m+s)!}{s!} \frac{r^{2s}}{(2jl)^{2m+2s+1}} P_{2s}(\cos\theta) . \tag{23}$$

The series appearing in expansions (22) and (23) in the subscript j are expressed by the Riemann ζ function:

$$G^0(\mathbf{x}) = \frac{1}{r} + 2 \sum_{s=0}^{\infty} \frac{r^{2s}}{(2l)^{2s+1}} \zeta(2s+1) P_{2s}(\cos\theta) , \tag{24}$$

$$G^{2m}(\mathbf{x}) = \frac{P_{2m}(\cos\theta)}{r^{2m+1}} + \frac{2}{m!} \sum_{s=0}^{\infty} \frac{(m+s)!}{s!} \frac{r^{2s}\zeta(2m+2s+1)}{(2l)^{2m+2s+1}} P_{2s}(\cos\theta). \tag{25}$$

Formulas (24) and (25) represent the sought expansions of the singular periodic solutions (7) and (8) in series in spherical functions suitable for $r < 2l$.

4. Investigation of an Infinite System of Linear Algebraic Equations. As has been shown in Sec. 2, the function (9) completely satisfies the Laplace equation (1), the periodicity conditions, and the asymptotic condition at infinity (2). Let us select the coefficients B_0, B_1, ... so that the function (3) also satisfies the homogeneous Dirichlet boundary condition (1). Thus, according to formulas (24) and (25), for the function (9) we have the expansion

$$e(\varepsilon; \mathbf{x}) = B_0 + \frac{l}{r} + \sum_{m=1}^{\infty} \frac{B_m}{2^{2m}} \zeta(2m+1) + \sum_{s=1}^{\infty} \frac{r^{2s}}{(2l)^{2s}} \zeta(2s+1) P_{2s}(\cos\theta) +$$

$$+\sum_{s=1}^{\infty} B_s \frac{l^{2s+1}}{r^{2s+1}} P_{2s}(\cos\theta) + \sum_{s=1}^{\infty}\sum_{m=1}^{\infty} \frac{B_m}{m!}\frac{(m+s)!}{2^{2m}s!}\frac{r^{2s}}{(2l)^{2s}}\zeta(2m+2s+1) P_{2s}(\cos\theta)\,. \tag{26}$$

We equate the right-hand side of relation (26) to zero when $r = \varepsilon l$. Starting from the linear independence of the Legendre polynomials, to determine the coefficients B_1, B_2, ... we derive the following infinite system of linear equations (s = 1, 2, ...):

$$B_s = -\sum_{m=1}^{\infty} \frac{B_m}{m!}\frac{(m+s)!}{2^{2m+2s}s!}\zeta(2m+2s+1)\,\varepsilon^{4s+1} - \frac{\varepsilon^{4s+1}}{2^{2s}}\zeta(2s+1)\,. \tag{27}$$

The coefficient B_0 is determined by the equality

$$B_0 = -\frac{1}{\varepsilon} - \sum_{m=1}^{\infty} \frac{B_m}{2^{2m}}\zeta(2m+1)\,. \tag{28}$$

We introduce the notation

$$c_{sm} = -\frac{1}{2^{2s}s!}\frac{(m+s)!}{m!2^{2m}}\zeta(2m+2s+1) \tag{29}$$

and evaluate the sum

$$C_s = \sum_{m=1}^{\infty} |c_{sm}|\,.$$

Since $\zeta(2m+2s+1)$ is less than $\zeta(2s+1)$, we have

$$C_s < \frac{\zeta(2s+1)}{2^{2s}s!}\sum_{m=1}^{\infty}(m+1)(m+2)\ldots(m+s)\,4^{-m}\,. \tag{30}$$

We sum up the series on the right-hand side of inequality (30). Using the well-known equation $(1-x)^{-1}$ = $1 + x + x^2 + \ldots$, we obtain

$$\sum_{m=1}^{\infty}(m+s)(m+s-1)\ldots(m+1)\,x^m = \frac{d^s}{dx^s}\left(\frac{x^s}{1-x}\right) \quad (|x| < 1)\,.$$

Since the integral part of the fraction $x^s(1-x)^{-1}$ represents a polynomial of the $(s-1)$th degree, we confirm the identity

$$\frac{d^s}{dx^s}\left(\frac{x^s}{1-x}\right) = \frac{s!}{(1-x)^{s+1}}$$

and establish

$$\sum_{m=1}^{\infty}\frac{(m+s)!}{m!}4^{-m} = \left[\left(\frac{4}{3}\right)^{s+1} - 1\right]s!\,.$$

Substituting this expression into the right-hand side of inequality (30), we derive

$$C_s < 4\zeta\,(2s+1)\left(\frac{1}{3^{s+1}} - \frac{1}{4^{s+1}}\right). \tag{31}$$

Finally, taking into account the evident estimate

$$\sum_{k=2}^{\infty} \frac{1}{k^m} < \int_1^{\infty} \frac{dt}{t^m} = \frac{1}{m-1}\,,$$

or, which is the same, $\zeta(m) < 1 + (m-1)^{-1}$, we obtain $\zeta(2s+1) < 3/2$ for $s \geq 1$. In doing so, from the inequality (31) we derive the following result:

$$C_s < \frac{2}{3^s}\,. \tag{32}$$

It is easily seen that inequality (32) yields $C_s < 2/3$. This means that the system of equations (27) is quite regular (see, in particular, [6], Chapter 1, Sec. 2) for any value of $\varepsilon \in (0, 1]$. The approximate solution of system (27) (in defect) can be obtained by the reduction method.

5. Asymptotics of the Reduced Logarithmic Capacity. According to formulas (17) and (28), we have

$$c_{\log}\,(\overline{G}_\varepsilon) = 4l \exp\left\{-\frac{1}{\varepsilon} - \mathbf{C} - \sum_{m=1}^{\infty} \frac{B_m}{2^{2m}}\,\zeta\,(2m+1)\right\}, \tag{33}$$

where B_1, B_2, ... is the solution of the infinite system of linear equations (27). It is easily seen that

$$B_m \sim -\frac{\varepsilon^{4m+1}}{2^{2m}}\,\zeta\,(2m+1)\,, \quad \varepsilon \to 0\,. \tag{34}$$

Restricting ourselves to only one equation in system (27), we obtain

$$B_1 = -\frac{\varepsilon^5 2^{-2} \zeta\,(3)}{1 + \varepsilon^5 2^{-3} \zeta\,(5)}\,. \tag{35}$$

Substitution of expression (34) for $m = 1$ into formula (33) yields

$$c_{\log}\,(\overline{G}_\varepsilon) = 4l \exp\left\{-\varepsilon^{-1} - \mathbf{C}\right\}\,(1 + \varepsilon^5 2^{-4} \zeta\,(3)^2 + O\,(\varepsilon^6))\,. \tag{36}$$

It is noteworthy that the use of rational-fractional approximations of the type (35) improves the exactness of the asymptotic formula (36) (see, for example, [7, 8]).

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NOTATION

$\mathbb{B}_l(O)$, sphere of radius l with center at the point O; $\mathbb{R}^3$, space; $\mathbb{R}^2$, plane; $\mathbf{x} = (\mathbf{y}, z)$, Cartesian coordinates in the space; $\mathbf{y} = (y_1, y_2)$, Cartesian coordinates on the plane; $\mathbf{C}$, Euler constant; ε, positive small parameter; G_ε, spherical inclusion; $c_{\log}(\overline{G}_\varepsilon)$, reduced logarithmic capacity; $e(\varepsilon;\, x)$, temperature; εl, radius of the spherical inclusion G_ε; g_ε, boundary of the spherical inclusion G_ε; $G^{2m}(\mathbf{x})$, singular periodic harmonic functions; $K_m(x)$, Macdonald functions, $2l$, distance between neighboring inclusions; $P_n(\mu)$, Legendre polynomial; Q_ε, bed of thickness $2l$ with hole G_ε; $\zeta(x)$, Riemann ζ function; $\omega_1(t)$, "sawtooth" function; B_m, undetermined coefficients. Subscripts and superscripts: m, s = 1, 2, ...; log, logarithmic.

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